## 2007-2008 Домашнее задание семинара m3-02

### Тема: Предел функции нескольких переменных

**9.** Найдите предел по совокупности переменных $\lim_{(x,y)\to(a,b)} u(x,y)$, если

**С** **(1)** $u(x,y)=xy,\ a=2,\ b=3,$

**Д** **(2)** $u(x,y)=\sin(x+y),\ a=\pi,\ b=\pi,$

**Э** **(3)** $u(x,y)=e^{xy},\ a=2,\ b=3,$ **(4)** $u(x,y)=\ln(xy),\ a=2,\ b=\frac{1}{2},$

**10.** Найдите предел по совокупности переменных $\lim_{(x,y)\to(a,b)} u(x,y)$, если

**С** **(1)** $u(x,y)=\frac{x^3+y^3}{x^2+y^2},\ a=0,\ b=0,$ **(2)** $u(x,y)=xy\ln(x^2+y^2),\ a=0,\ b=0,$

**Д** **(3)** $u(x,y)=\frac{x^4+y^4}{x^2+y^2},\ a=0,\ b=0,$ **(4)** $u(x,y)=y\ln(x^2+y^2),\ a=0,\ b=0,$

**Э** **(5)** $u(x,y)=\frac{x^6+y^6}{x^4+y^4},\ a=0,\ b=0,$ **(6)** $u(x,y)=x\ln(x^2+y^2),\ a=0,\ b=0,$

**(7)** $u(x,y)=x\ln y,\ a=0,\ b=1,$ **(8)** $u(x,y)=x\ln(xy),\ a=1,\ b=1,$

**(9)** $u(x,y)=xy\ln(xy),\ a=1,\ b=1,$ **(10)** $u(x,y)=(x^2+y^2)\ln(x^2+y^2),\ a=0,\ b=0,$

**(11)** $u(x,y)=\frac{\ln(1+x^2+y^2)}{x^2+y^2},\ a=0,\ b=0,$ **(12)** $u(x,y)=\frac{x^2+y^2}{\ln(x^2+y^2)},\ a=0,\ b=0,$

**(13)** $u(x,y)=\frac{e^{x^2+y^2}-1}{x^2+y^2},\ a=0,\ b=0,$ **(14)** $u(x,y)=\frac{\sin(x^2+y^2)}{x^2+y^2},\ a=0,\ b=0,$

**(15)** $u(x,y)=(x^2+y^2)\sin\frac{1}{x^2+y^2},\ a=0,\ b=0,$

**(16)** $u(x,y)=x\sin\frac{1}{y}+y\sin\frac{1}{x},\ u(x,0)=0,\ u(0,y)=0,\ a=0,\ b=0.$

**11.** Найдите $\lim_{(x;y)\to(0;0)} u(x,y)$, если точка $(x;y)$ приближается к точке $(0;0)$ по кривой $x=\varphi(t)$, $y=\psi(t)$, $t\to 0$,

**С** **(1)** $u(x,y)=\frac{2xy}{x^2+y^2},$ **(a)** $\varphi(t)=t,\ \psi(t)=t,$ **(b)** $\varphi(t)=t,\ \psi(t)=-t,$ **(c)** $\varphi(t)=t,\ \psi(t)=2t,$

**(2)** $u(x,y)=\frac{2x^3y}{x^6+y^2},$ **(a)** $\varphi(t)=t,\ \psi(t)=t,$ **(b)** $\varphi(t)=t,\ \psi(t)=t^2,$ **(c)** $\varphi(t)=t,\ \psi(t)=t^3,$ **(d)** $\varphi(t)=t,\ \psi(t)=t^4,$

**Д** **(3)** $u(x,y)=\frac{x^2-y^2}{x^2+y^2},$ **(a)** $\varphi(t)=2t,\ \psi(t)=t,$ **(b)** $\varphi(t)=-2t,\ \psi(t)=t,$ **(c)** $\varphi(t)=t,\ \psi(t)=2t,$

**(4)** $u(x,y)=\frac{2xy^2}{x^2+y^4},$ **(a)** $\varphi(t)=t,\ \psi(t)=t,$ **(b)** $\varphi(t)=t^2,\ \psi(t)=t,$ **(c)** $\varphi(t)=-t^2,\ \psi(t)=t,$

**Э** **(5)** $u(x,y)=\frac{x^4-y^4}{x^4+y^4},$ **(a)** $\varphi(t)=2t,\ \psi(t)=t,$ **(b)** $\varphi(t)=-2t,\ \psi(t)=t,$ **(c)** $\varphi(t)=t,\ \psi(t)=2t,$

**(6)** $u(x,y)=\frac{\sin(2xy)}{x^2+y^2},$ **(a)** $\varphi(t)=t,\ \psi(t)=t,$ **(b)** $\varphi(t)=-t,\ \psi(t)=t,$ **(c)** $\varphi(t)=t,\ \psi(t)=2t,$

**(7)** $u(x,y)=\frac{2x^2y}{x^4+y^2},$ **(a)** $\varphi(t)=t,\ \psi(t)=t,$ **(b)** $\varphi(t)=t,\ \psi(t)=t^2,$ **(c)** $\varphi(t)=t,\ \psi(t)=-t^2,$

**(8)** $u(x,y)=\frac{2x^3y}{x^6+y^2},$ **(a)** $\varphi(t)=t,\ \psi(t)=t,$ **(b)** $\varphi(t)=t,\ \psi(t)=t^2,$ **(c)** $\varphi(t)=t,\ \psi(t)=t^3,$

**12.** Докажите, что предел по совокупности переменных $\lim_{(x,y)\to(a,b)} u(x,y)$, не существует, если

**С** **(1)** $u(x,y)=\frac{x^2-y^2}{x^2+y^2},\ a=0,\ b=0,$ **(2)** $u(x,y)=\frac{x^4-y^2}{x^4+y^2},\ a=0,\ b=0,$

**Д** **(3)** $u(x,y)=\frac{2xy}{x^2+y^2},\ a=0,\ b=0,$ **(4)** $u(x,y)=\frac{2x^2y}{x^4+y^2},\ a=0,\ b=0,$

**Э** **(5)** $u(x,y)=\frac{2xy^2}{x^2+y^4},\ a=0,\ b=0,$ **(6)** $u(x,y)=\frac{x^2}{x^2+y^2},\ a=0,\ b=0,$

**(7)** $u(x,y)=\frac{x^2+y^2}{x^4+y^4},\ a=0,\ b=0,$ **(8)** $u(x,y)=\frac{x^2+y^2}{x^2y^2+(x-y)^2},\ a=0,\ b=0,$

**(9)** $u(x,y)=\ln(x^2+y^2),\ a=0,\ b=0,$ **(10)** $u(x,y)=\frac{\ln(x^2+y^2)}{x^2+y^2},\ a=0,\ b=0,$

**(11)** $u(x,y)=\frac{e^{x^2+y^2}}{x^2+y^2},\ a=0,\ b=0,$ **(12)** $u(x,y)=\frac{\sin(xy)}{x^2+xy+y^2},\ a=0,\ b=0,$

**(13)** $u(x,y)=\frac{\sin(xy)}{x^2+y^2},\ a=0,\ b=0,$ **(14)** $u(x,y)=\frac{\sin(x^2+y^2)}{x^2+y^2},\ a=0,\ b=0,$

**(15)** $u(x,y)=\sin\frac{1}{x^2+y^2},\ a=0,\ b=0,$ **(16)** $u(x,y)=\sin\frac{1}{y}+\sin\frac{1}{x},\ a=0,\ b=0,$

**13.** Найдите предел в бесконечно удаленной точке или докажите, что предел не существует,

**С** **(1)** $u(x,y)=xe^{-x}+ye^{-y}$, **(2)** $u(x,y)=\frac{x^2+y^2}{x^4+y^4}$, **(3)** $u(x,y)=\frac{x^2-y^2}{x^2+y^2}$,

**Д** **(4)** $u(x,y)=e^{-x}+e^{-y}$, **(5)** $u(x,y)=\frac{x+y}{x^2+xy+y^2}$, **(6)** $u(x,y)=\frac{x^2y^2}{x^4+y^4}$,

**Т** **(7)** $u(x,y)=ye^{-x}+xe^{-y}$, **(8)** $u(x,y)=\frac{x^3+y^3}{x^4+y^4}$, **(9)** $u(x,y)=\frac{x^2+y^2}{x^2+y^4}$, **(10)** $u(x,y)=\frac{xy}{x^2+xy+y^2}$,

**(11)** $u(x,y)=xye^{-x^2-y^2}$, **(12)** $u(x,y)=e^{-x^2}$, **(13)** $u(x,y)=ye^{-x^2}$, **(14)** $u(x,y)=\frac{\ln(x^2+y^2)}{x^2+y^2}$,

**(15)** $u(x,y)=(x^2+y^2)\sin\frac{1}{x^2+y^2}$, **(16)** $u(x,y)=(x^2+y^2)\operatorname{tg}\frac{1}{x^2+y^2}$,

**(17)** $u(x,y)=(x^2+y^2)\arcsin\frac{1}{x^2+y^2}$, **(18)** $u(x,y)=(x^2+y^2)\operatorname{arctg}\frac{1}{x^2+y^2}$,