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for seminar

1 -st kind

direct computation

$$\int_1^{\infty} x^{-\frac{1}{2}}\,dx \to \infty \qquad \int_1^{\infty} x^{-\frac{3}{2}}\,dx \to 2$$

$$\int_1^{\infty} x^{-2}\,dx \to 1 \qquad \int_1^{\infty} x^{-3}\,dx \to \frac{1}{2} \qquad \int_1^{\infty} x^{-4}\,dx \to \frac{1}{3} \qquad \int_1^{\infty} x^{-5}\,dx \to \frac{1}{4}$$

by parts

$$\int_1^{\infty} x^{-2}\cdot\ln(x)\,dx \to 1 \qquad \int_1^{\infty} x^{-2}\cdot\ln(x)^2\,dx \to 2 \qquad \int_1^{\infty} x^{-2}\cdot\ln(x)^3\,dx \to 6 \qquad \int_1^{\infty} x^{-2}\cdot\ln(x)^4\,dx \to 24$$

$$\int_1^{\infty} x^{-4}\cdot\ln(x)\,dx \to \frac{1}{9} \qquad \int_1^{\infty} x^{-4}\cdot\ln(x)^2\,dx \to \frac{2}{27} \qquad \int_1^{\infty} x^{-4}\cdot\ln(x)^3\,dx \to \frac{2}{27} \qquad \int_1^{\infty} x^{-4}\cdot\ln(x)^3\,dx \to \frac{2}{27}$$

$$\int_0^{\infty} x\cdot e^{-x}\,dx \to 1 \qquad \int_0^{\infty} x^2\cdot e^{-x}\,dx \to 2 \qquad \int_0^{\infty} x^3\cdot e^{-x}\,dx \to 6 \qquad \int_0^{\infty} x^4\cdot e^{-x}\,dx \to 24$$

direct

$$\int_0^{\infty} \frac{1}{1+x^2}\,dx \to \frac{1}{2}\cdot\pi \qquad \int_0^{\infty} \frac{1}{4+x^2}\,dx \to \frac{1}{4}\cdot\pi \qquad \int_0^{\infty} \frac{x}{1+x^4}\,dx \to \frac{1}{4}\cdot\pi$$

by parts

$$\int_0^{\infty} \frac{1}{\left(1+x^2\right)^2}\,dx \to \frac{1}{4}\cdot\pi \qquad \int_0^{\infty} \frac{1}{\left(1+x^2\right)^3}\,dx \to \frac{3}{16}\cdot\pi$$

$$\int_0^{\infty} e^{-x}\cdot\sin(x)\,dx \to \frac{1}{2} \qquad \int_0^{\infty} e^{-x}\cdot\cos(x)\,dx \to \frac{1}{2}$$

$$\int_1^{10000} \frac{\sin(\ln(x))}{x^2}\,dx \to \frac{1}{20000}\cdot\frac{-1+\tan(2\cdot\ln(2)+2\cdot\ln(5))^2-2\cdot\tan(2\cdot\ln(2)+2\cdot\ln(5))}{1+\tan(2\cdot\ln(2)+2\cdot\ln(5))^2}+\frac{1}{2} = 0.500038214914828$$

$$\int_1^{10000} \frac{\sin(\ln(x))}{x^4}\,dx \to \frac{1}{10000000000000}\cdot\frac{-1-6\cdot\tan(2\cdot\ln(2)+2\cdot\ln(5))+\tan(2\cdot\ln(2)+2\cdot\ln(5))^2}{1+\tan(2\cdot\ln(2)+2\cdot\ln(5))^2}+\frac{1}{10} = 0.10000000000$$

split to simple fractions and compute antiderivative

$$\int_0^{\infty} \frac{1}{x^2+4x+3}\,dx \to \frac{1}{2}\cdot\ln(3) \qquad \int_0^{\infty} \frac{1}{(x+1)\cdot(x+2)\cdot(x+3)}\,dx \to \ln(2)-\frac{1}{2}\cdot\ln(3)$$

change variable

$$\int_0^{\infty} x\cdot e^{\frac{-x^2}{2}}\,dx \to 1$$

$$\int_0^{\infty} x\cdot e^{-x^2}\,dx \to \frac{1}{2} \qquad \int_0^{\infty} x^2\cdot e^{-x^3}\,dx \to \frac{1}{3} \qquad \int_0^{\infty} x^3\cdot e^{-x^4}\,dx \to \frac{1}{4}$$

$$\int_0^{\infty} x^3\cdot e^{-x^2}\,dx \to \frac{1}{2} \qquad \int_0^{\infty} x^8\cdot e^{-x^3}\,dx \to \frac{2}{3} \qquad \int_0^{\infty} x^{19}\cdot e^{-x^4}\,dx \to 6$$

Let Known $\int_0^{\infty} e^{-x^2}\,dx \to \frac{1}{2}\cdot\pi^{\frac{1}{2}}$ $\int_{-\infty}^{\infty} e^{-x^2}\,dx \to \pi^{\frac{1}{2}}$

by parts

$$\int_0^{\infty} x^2\cdot e^{-x^2}\,dx \to \frac{1}{4}\cdot\pi^{\frac{1}{2}} \qquad \int_0^{\infty} x^4\cdot e^{-x^2}\,dx \to \frac{3}{8}\cdot\pi^{\frac{1}{2}} \qquad \int_0^{\infty} x^6\cdot e^{-x^2}\,dx \to \frac{15}{16}\cdot\pi^{\frac{1}{2}}$$

$$\int_0^{\infty} x\cdot e^{-x^4}\,dx \to \frac{1}{4}\cdot\pi^{\frac{1}{2}} \qquad \int_0^{\infty} x^9\cdot e^{-x^4}\,dx \to \frac{3}{16}\cdot\pi^{\frac{1}{2}}$$

2 -nd kind

direct

$$\int_0^{1} x^{-\frac{1}{2}}\,dx \to 2 \qquad \int_0^{1} x^{-\frac{3}{2}}\,dx \to \infty \qquad \int_0^{1} \frac{1}{\sqrt{1-x}}\,dx \to 2 \qquad \int_0^{1} \frac{x}{\sqrt{1-x}}\,dx \to \frac{4}{3}$$

by parts

$$\int_0^1 \ln(x)\,dx \to -1 \qquad \int_0^1 x\cdot\ln(x)\,dx \to \frac{-1}{4} \qquad \int_0^1 x^2\cdot\ln(x)\,dx \to \frac{-1}{9} \qquad \int_0^1 x^3\cdot\ln(x)\,dx \to \frac{-1}{16}$$

direct

change variable

$$\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx \to \frac{1}{2}\cdot\pi \qquad \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx \to 1$$

$$\int_0^1 \frac{x^2}{\sqrt{1-x^2}}\,dx \to \frac{1}{4}\cdot\pi \qquad \int_0^1 \frac{x^3}{\sqrt{1-x^2}}\,dx \to \frac{2}{3}$$

$$\int_0^1 \frac{x}{\sqrt{1-x^4}}\,dx \to \frac{1}{4}\cdot\pi \qquad \int_0^1 \frac{x^3}{\sqrt{1-x^4}}\,dx \to \frac{1}{2}$$

$$\int_0^1 \frac{1}{\sqrt{x-x^2}}\,dx \to \pi$$

$$\int_0^{\frac{\pi}{2}} \cos(x)\cdot\ln(\sin(x))\,dx \to -1 \qquad \int_0^{\frac{\pi}{2}} \sin(x)\cdot\cos(x)\cdot\ln(\sin(x))\,dx \to \frac{-1}{4} \qquad \int_0^{\frac{\pi}{2}} \sin(x)^3\cdot\cos(x)\cdot\ln(\sin(x))\,dx \to \frac{-1}{16}$$

$$\int_0^{\frac{\pi}{2}} \cos(x)^3\cdot\ln(\sin(x))\,dx \to \frac{-8}{9}$$

0034