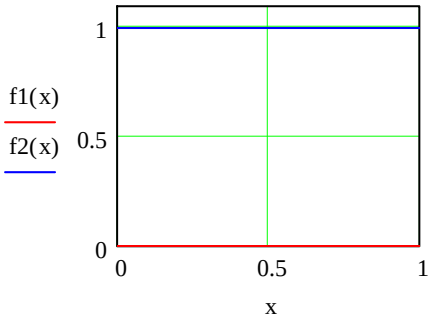
C Alexey A Bykov, 16 Feb 2008.
Moscow State University, Department of physics, boombook@yandex.ru, boombook.narod.ru

Figure a<x<b, y1(x)<y<y2(x).

1. Cylinder

$f1(x) := 0$ $f2(x) := 1$ $a := 0$ $b := 1$



$$M1 := \int_a^b 1\cdot(f2(x) - f1(x))\,dx \qquad M1 \to 1$$

$$Mx := \int_a^b x\cdot(f2(x) - f1(x))\,dx \qquad Mx \to \frac{1}{2} \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to \frac{1}{2}$$

$$Mxx := \int_a^b x^2\cdot(f2(x) - f1(x))\,dx \qquad Mxx \to \frac{1}{3} \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{1}{3}$$

$$D := xx_{average} - x_{average}^2 \qquad D \to \frac{1}{12}$$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\,dx \qquad My \to \frac{1}{2} \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{1}{2}$$

$$VOXM1 := \int_a^b \pi\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXM1 \to \pi$$

$$VOXMx := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXMx \to \frac{1}{2}\cdot\pi \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to \frac{1}{2}$$

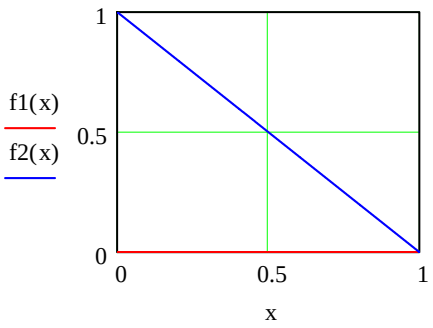
$$VOYM1 := \int_a^b 2\pi\cdot x\cdot(f2(x) - f1(x))\,dx \qquad VOYM1 \to \pi$$

$$VOYMy := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOYMy \to \frac{1}{2}\cdot\pi \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{1}{2}$$

Figure a<x<b, y1(x)<y<y2(x).

2. Cone

$f1(x) := 0 \qquad f2(x) := 1 - x \qquad a := 0 \qquad b := 1$



$$M1 := \int_a^b 1 \cdot (f2(x) - f1(x))\, dx \qquad M1 \to \frac{1}{2}$$

$$Mx := \int_a^b x \cdot (f2(x) - f1(x))\, dx \qquad Mx \to \frac{1}{6} \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to \frac{1}{3}$$

$$Mxx := \int_a^b x^2 \cdot (f2(x) - f1(x))\, dx \qquad Mxx \to \frac{1}{12} \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{1}{6}$$

$$D := xx_{average} - x_{average}^2 \qquad D \to \frac{1}{18}$$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\, dx \qquad My \to \frac{1}{6} \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{1}{3}$$

$$VOXM1 := \int_a^b \pi \cdot \left(f2(x)^2 - f1(x)^2\right) dx \qquad VOXM1 \to \frac{1}{3} \cdot \pi$$

$$VOXMx := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx \qquad VOXMx \to \frac{1}{12} \cdot \pi \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to \frac{1}{4}$$

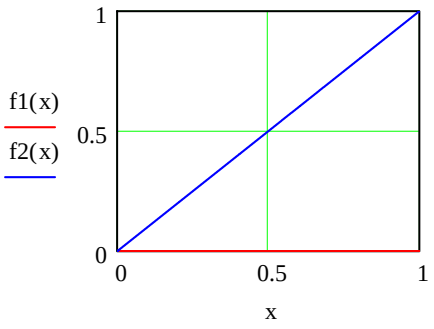
$$VOYM1 := \int_a^b 2\pi \cdot x \cdot (f2(x) - f1(x))\, dx \qquad VOYM1 \to \frac{1}{3} \cdot \pi$$

$$VOYMy := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx \qquad VOYMy \to \frac{1}{12} \cdot \pi \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{1}{4}$$

Figure a<x<b, y1(x)<y<y2(x).

3. Inverse cone

$f1(x) := 0$ $f2(x) := x$ $a := 0$ $b := 1$



$$M1 := \int_a^b 1\cdot(f2(x) - f1(x))\,dx \qquad M1 \to \frac{1}{2}$$

$$Mx := \int_a^b x\cdot(f2(x) - f1(x))\,dx \qquad Mx \to \frac{1}{3} \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to \frac{2}{3}$$

$$Mxx := \int_a^b x^2\cdot(f2(x) - f1(x))\,dx \qquad Mxx \to \frac{1}{4} \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{1}{2}$$

$$D := xx_{average} - x_{average}^{\,2} \qquad D \to \frac{1}{18}$$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\,dx \qquad My \to \frac{1}{6} \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{1}{3}$$

$$VOXM1 := \int_a^b \pi\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXM1 \to \frac{1}{3}\cdot\pi$$

$$VOXMx := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXMx \to \frac{1}{4}\cdot\pi \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to \frac{3}{4}$$

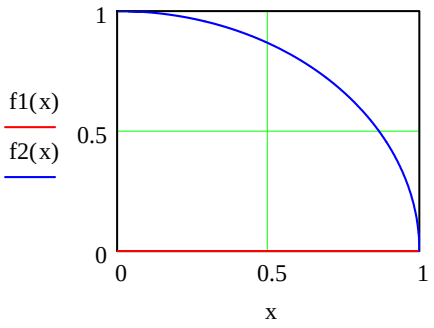
$$VOYM1 := \int_a^b 2\pi\cdot x\cdot(f2(x) - f1(x))\,dx \qquad VOYM1 \to \frac{2}{3}\cdot\pi$$

$$VOYMy := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOYMy \to \frac{1}{4}\cdot\pi \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{3}{8}$$

Figure a<x<b, y1(x)<y<y2(x).

4. Sphere

$f1(x) := 0$ $f2(x) := \sqrt{1 - x^2}$ $a := 0$ $b := 1$



$$M1 := \int_a^b 1 \cdot (f2(x) - f1(x))\, dx \qquad M1 \to \frac{1}{4} \cdot \pi$$

$$Mx := \int_a^b x \cdot (f2(x) - f1(x))\, dx \qquad Mx \to \frac{1}{3} \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to \frac{4}{3 \cdot \pi}$$

$$Mxx := \int_a^b x^2 \cdot (f2(x) - f1(x))\, dx \qquad Mxx \to \frac{1}{16} \cdot \pi \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{1}{4}$$

$$D := xx_{average} - x_{average}^2 \qquad D \to \frac{1}{4} - \frac{16}{9 \cdot \pi^2}$$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\, dx \qquad My \to \frac{1}{3} \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{4}{3 \cdot \pi}$$

$$VOXM1 := \int_a^b \pi \cdot \left(f2(x)^2 - f1(x)^2\right) dx \qquad VOXM1 \to \frac{2}{3} \cdot \pi$$

$$VOXMx := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx \qquad VOXMx \to \frac{1}{4} \cdot \pi \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to \frac{3}{8}$$

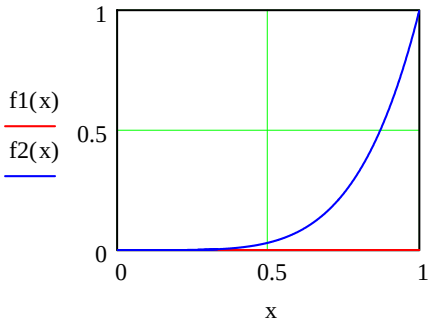
$$VOYM1 := \int_a^b 2\pi \cdot x \cdot (f2(x) - f1(x))\, dx \qquad VOYM1 \to \frac{2}{3} \cdot \pi$$

$$VOYMy := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx \qquad VOYMy \to \frac{1}{4} \cdot \pi \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{3}{8}$$

Figure a<x<b, y1(x)<y<y2(x).

5. Power 5

$f1(x) := 0$ $f2(x) := x^5$ $a := 0$ $b := 1$



$$M1 := \int_a^b 1\cdot(f2(x) - f1(x))\,dx \qquad M1 \to \frac{1}{6}$$

$$Mx := \int_a^b x\cdot(f2(x) - f1(x))\,dx \qquad Mx \to \frac{1}{7} \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to \frac{6}{7}$$

$$Mxx := \int_a^b x^2\cdot(f2(x) - f1(x))\,dx \qquad Mxx \to \frac{1}{8} \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{3}{4}$$

$$D := xx_{average} - x_{average}^2 \qquad D \to \frac{3}{196}$$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\,dx \qquad My \to \frac{1}{22} \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{3}{11}$$

$$VOXM1 := \int_a^b \pi\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXM1 \to \frac{1}{11}\cdot\pi$$

$$VOXMx := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXMx \to \frac{1}{12}\cdot\pi \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to \frac{11}{12}$$

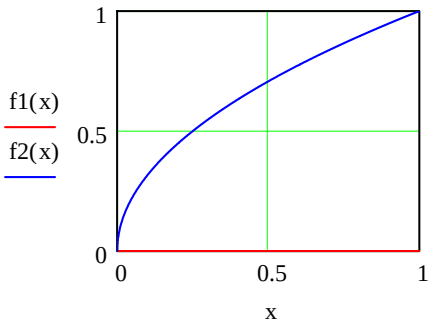
$$VOYM1 := \int_a^b 2\pi\cdot x\cdot(f2(x) - f1(x))\,dx \qquad VOYM1 \to \frac{2}{7}\cdot\pi$$

$$VOYMy := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOYMy \to \frac{1}{12}\cdot\pi \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{7}{24}$$

Figure a<x<b, y1(x)<y<y2(x).

6. root

$f1(x) := 0 \qquad f2(x) := \sqrt{x} \qquad a := 0 \qquad b := 1$



$$M1 := \int_a^b 1\cdot(f2(x) - f1(x))\,dx \qquad M1 \to \frac{2}{3}$$

$$Mx := \int_a^b x\cdot(f2(x) - f1(x))\,dx \qquad Mx \to \frac{2}{5} \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to \frac{3}{5}$$

$$Mxx := \int_a^b x^2\cdot(f2(x) - f1(x))\,dx \qquad Mxx \to \frac{2}{7} \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{3}{7}$$

$$D := xx_{average} - x_{average}^2 \qquad D \to \frac{12}{175}$$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\,dx \qquad My \to \frac{1}{4} \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{3}{8}$$

$$VOXM1 := \int_a^b \pi\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXM1 \to \frac{1}{2}\cdot\pi$$

$$VOXMx := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXMx \to \frac{1}{3}\cdot\pi \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to \frac{2}{3}$$

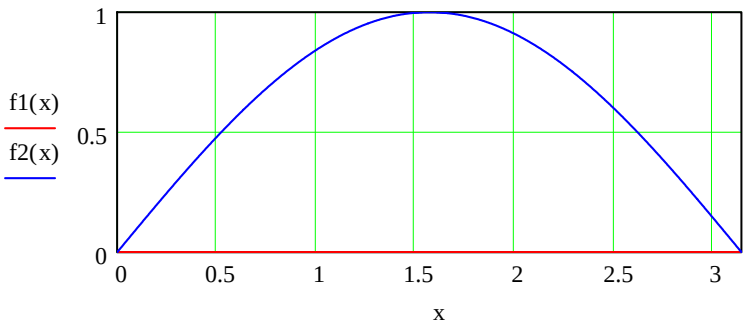
$$VOYM1 := \int_a^b 2\pi\cdot x\cdot(f2(x) - f1(x))\,dx \qquad VOYM1 \to \frac{4}{5}\cdot\pi$$

$$VOYMy := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOYMy \to \frac{1}{3}\cdot\pi \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{5}{12}$$

Figure a<x<b, y1(x)<y<y2(x).

7. Sin

$f1(x) := 0$ $f2(x) := \sin(x)$ $a := 0$ $b := \pi$



$$M1 := \int_a^b 1\cdot(f2(x) - f1(x))\,dx \qquad M1 \to 2$$

$$Mx := \int_a^b x\cdot(f2(x) - f1(x))\,dx \qquad Mx \to \pi \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to \frac{1}{2}\cdot\pi$$

$$Mxx := \int_a^b x^2\cdot(f2(x) - f1(x))\,dx \qquad Mxx \to \pi^2 - 4 \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{1}{2}\cdot\pi^2 - 2$$

$$D := xx_{average} - x_{average}^2 \qquad D \to \frac{1}{4}\cdot\pi^2 - 2$$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\,dx \qquad My \to \frac{1}{4}\cdot\pi \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{1}{8}\cdot\pi$$

$$VOXM1 := \int_a^b \pi\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXM1 \to \frac{1}{2}\cdot\pi^2$$

$$VOXMx := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXMx \to \frac{1}{4}\cdot\pi^3 \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to \frac{1}{2}\cdot\pi$$

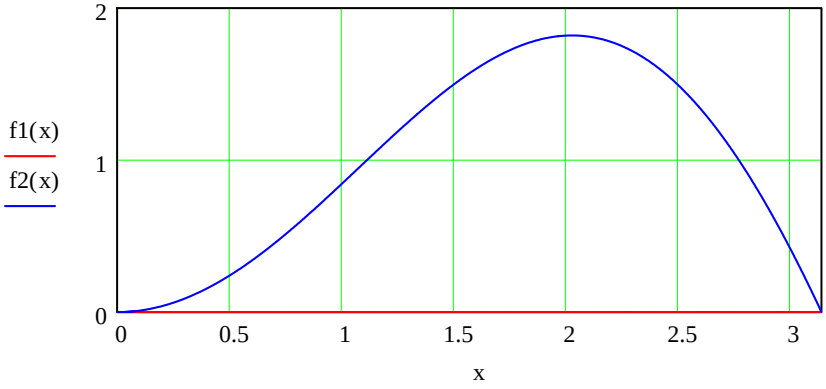
$$VOYM1 := \int_a^b 2\pi\cdot x\cdot(f2(x) - f1(x))\,dx \qquad VOYM1 \to 2\cdot\pi^2$$

$$VOYMy := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOYMy \to \frac{1}{4}\cdot\pi^3 \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{1}{8}\cdot\pi$$

Figure a<x<b, y1(x)<y<y2(x).

8. xsin

$f1(x) := 0 \qquad f2(x) := x \cdot \sin(x) \qquad a := 0 \qquad b := \pi$



$$M1 := \int_a^b 1 \cdot (f2(x) - f1(x))\, dx \qquad M1 \to \pi$$

$$Mx := \int_a^b x \cdot (f2(x) - f1(x))\, dx \qquad Mx \to \pi^2 - 4 \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to \frac{\pi^2 - 4}{\pi}$$

$$Mxx := \int_a^b x^2 \cdot (f2(x) - f1(x))\, dx \qquad Mxx \to \pi^3 - 6 \cdot \pi \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{\pi^3 - 6 \cdot \pi}{\pi}$$

$$D := xx_{average} - x_{average}^2 \qquad D \to \frac{\pi^3 - 6 \cdot \pi}{\pi} - \frac{\left(\pi^2 - 4\right)^2}{\pi^2}$$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\, dx \qquad My \to \frac{1}{12} \cdot \pi^3 - \frac{1}{8} \cdot \pi \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{\frac{1}{12} \cdot \pi^3 - \frac{1}{8} \cdot \pi}{\pi}$$

$$VOXM1 := \int_a^b \pi \cdot \left(f2(x)^2 - f1(x)^2\right) dx \qquad VOXM1 \to \frac{1}{6} \cdot \pi^4 - \frac{1}{4} \cdot \pi^2$$

$$VOXMx := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx \qquad VOXMx \to \frac{1}{8} \cdot \pi^5 - \frac{3}{8} \cdot \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to \frac{\frac{1}{8} \cdot \pi^5 - \frac{3}{8} \cdot \pi^3}{\frac{1}{6} \cdot \pi^4 - \frac{1}{4} \cdot \pi^2}$$

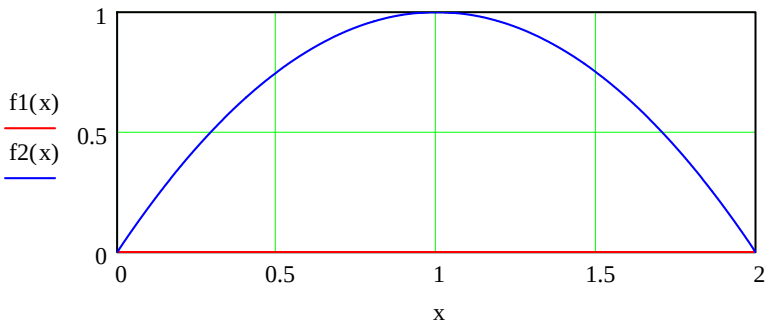
$$VOYM1 := \int_a^b 2\pi \cdot x \cdot (f2(x) - f1(x))\, dx \qquad VOYM1 \to 2 \cdot \pi^3 - 8 \cdot \pi$$

$$VOYMy := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx \qquad VOYMy \to \frac{1}{8} \cdot \pi^5 - \frac{3}{8} \cdot \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{\frac{1}{8} \cdot \pi^5 - \frac{3}{8} \cdot \pi^3}{2 \cdot \pi^3 - 8 \cdot \pi}$$

Figure a<x<b, y1(x)<y<y2(x).

9. parabola

$f1(x) := 0$ $f2(x) := x\cdot(2 - x)$ $a := 0$ $b := 2$



$$M1 := \int_a^b 1\cdot(f2(x) - f1(x))\,dx \qquad M1 \to \frac{4}{3}$$

$$Mx := \int_a^b x\cdot(f2(x) - f1(x))\,dx \qquad Mx \to \frac{4}{3} \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to 1$$

$$Mxx := \int_a^b x^2\cdot(f2(x) - f1(x))\,dx \qquad Mxx \to \frac{8}{5} \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{6}{5}$$

$$D := xx_{average} - x_{average}^2 \qquad D \to \frac{1}{5}$$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\,dx \qquad My \to \frac{8}{15} \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{2}{5}$$

$$VOXM1 := \int_a^b \pi\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXM1 \to \frac{16}{15}\cdot\pi$$

$$VOXMx := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXMx \to \frac{16}{15}\cdot\pi \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to 1$$

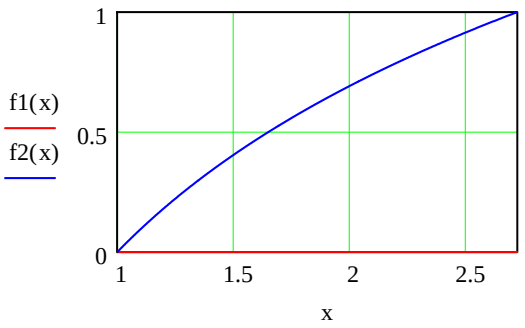
$$VOYM1 := \int_a^b 2\pi\cdot x\cdot(f2(x) - f1(x))\,dx \qquad VOYM1 \to \frac{8}{3}\cdot\pi$$

$$VOYMy := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOYMy \to \frac{16}{15}\cdot\pi \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{2}{5}$$

Figure a<x<b, y1(x)<y<y2(x).

10. logaruthm

$f1(x) := 0$ $f2(x) := \ln(x)$ $a := 1$ $b := e$



$$M1 := \int_a^b 1\cdot(f2(x) - f1(x))\,dx \qquad M1 \to 1$$

$$Mx := \int_a^b x\cdot(f2(x) - f1(x))\,dx \qquad Mx \to \frac{1}{4}\cdot\exp(2) + \frac{1}{4} \qquad x_{average} := \frac{Mx}{M1} \qquad x_{average} \to \frac{1}{4}\cdot\exp(2) + \frac{1}{4}$$

$$Mxx := \int_a^b x^2\cdot(f2(x) - f1(x))\,dx \qquad Mxx \to \frac{2}{9}\cdot\exp(3) + \frac{1}{9} \qquad xx_{average} := \frac{Mxx}{M1} \qquad xx_{average} \to \frac{2}{9}\cdot\exp(3) + \frac{1}{9}$$

$$D := xx_{average} - x_{average}^2 \qquad D \to \frac{2}{9}\cdot\exp(3) + \frac{1}{9} - \left($$

$$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\,dx \qquad My \to \frac{1}{2}\cdot\exp(1) - 1 \qquad y_{average} := \frac{My}{M1} \qquad y_{average} \to \frac{1}{2}\cdot\exp(1) - 1$$

$$VOXM1 := \int_a^b \pi\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXM1 \to \pi\cdot\exp(1) - 2\cdot\pi$$

$$VOXMx := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOXMx \to \frac{1}{4}\cdot\pi\cdot\exp(2 \qquad VOXx_{average} := \frac{VOXMx}{VOXM1} \qquad VOXx_{average} \to \frac{\frac{1}{4}\cdot\pi\cdot(}{\pi\cdot e}$$

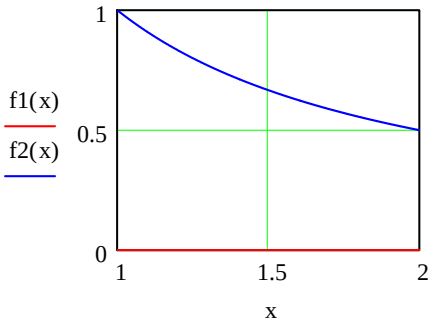
$$VOYM1 := \int_a^b 2\pi\cdot x\cdot(f2(x) - f1(x))\,dx \qquad VOYM1 \to \frac{1}{2}\cdot\pi\cdot\exp(2) + \frac{1}{2}\cdot\pi$$

$$VOYMy := \int_a^b \pi\cdot x\cdot\left(f2(x)^2 - f1(x)^2\right)dx \qquad VOYMy \to \frac{1}{4}\cdot\pi\cdot\exp(2 \qquad VOYy_{average} := \frac{VOYMy}{VOYM1} \qquad VOYy_{average} \to \frac{\frac{1}{4}\cdot\pi\cdot(}{\frac{1}{2}\cdot\pi\cdot(}$$

Figure a<x<b, y1(x)<y<y2(x).

11. Power -1

$f1(x) := 0$ $f2(x) := \frac{1}{x}$ $a := 1$ $b := 2$



$M1 := \int_a^b 1 \cdot (f2(x) - f1(x))\, dx$ $M1 \to \ln(2)$

$Mx := \int_a^b x \cdot (f2(x) - f1(x))\, dx$ $Mx \to 1$ $x_{average} := \frac{Mx}{M1}$ $x_{average} \to \frac{1}{\ln(2)}$

$Mxx := \int_a^b x^2 \cdot (f2(x) - f1(x))\, dx$ $Mxx \to \frac{3}{2}$ $xx_{average} := \frac{Mxx}{M1}$ $xx_{average} \to \frac{3}{2 \cdot \ln(2)}$

$D := xx_{average} - x_{average}^2$ $D \to \frac{3}{2 \cdot \ln(2)} - \frac{1}{\ln(2)^2}$

$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\, dx$ $My \to \frac{1}{4}$ $y_{average} := \frac{My}{M1}$ $y_{average} \to \frac{1}{4 \cdot \ln(2)}$

$VOXM1 := \int_a^b \pi \cdot \left(f2(x)^2 - f1(x)^2\right) dx$ $VOXM1 \to \frac{1}{2} \cdot \pi$

$VOXMx := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx$ $VOXMx \to \pi \cdot \ln(2)$ $VOXx_{average} := \frac{VOXMx}{VOXM1}$ $VOXx_{average} \to 2 \cdot \ln(2$

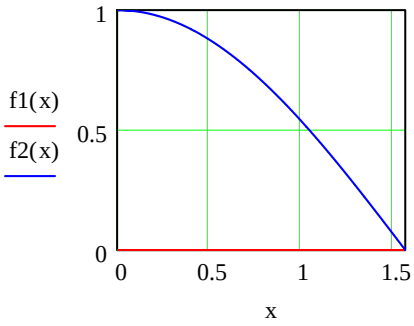
$VOYM1 := \int_a^b 2\pi \cdot x \cdot (f2(x) - f1(x))\, dx$ $VOYM1 \to 2 \cdot \pi$

$VOYMy := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx$ $VOYMy \to \pi \cdot \ln(2)$ $VOYy_{average} := \frac{VOYMy}{VOYM1}$ $VOYy_{average} \to \frac{1}{2} \cdot \ln($

Figure a<x<b, y1(x)<y<y2(x).

11. cosx

$f1(x) := 0$ $f2(x) := \cos(x)$ $a := 0$ $b := \frac{\pi}{2}$



$M1 := \int_a^b 1 \cdot (f2(x) - f1(x))\, dx$ $M1 \to 1$

$Mx := \int_a^b x \cdot (f2(x) - f1(x))\, dx$ $Mx \to \frac{1}{2} \cdot \pi - 1$ $x_{average} := \frac{Mx}{M1}$ $x_{average} \to \frac{1}{2} \cdot \pi - 1$

$Mxx := \int_a^b x^2 \cdot (f2(x) - f1(x))\, dx$ $Mxx \to \frac{1}{4} \cdot \pi^2 - 2$ $xx_{average} := \frac{Mxx}{M1}$ $xx_{average} \to \frac{1}{4} \cdot \pi^2 - 2$

$D := xx_{average} - x_{average}^2$ $D \to \frac{1}{4} \cdot \pi^2 - 2 - \left(\frac{1}{2} \cdot \pi -$

$My := \int_a^b \frac{\left(f2(x)^2 - f1(x)^2\right)}{2}\, dx$ $My \to \frac{1}{8} \cdot \pi$ $y_{average} := \frac{My}{M1}$ $y_{average} \to \frac{1}{8} \cdot \pi$

$VOXM1 := \int_a^b \pi \cdot \left(f2(x)^2 - f1(x)^2\right) dx$ $VOXM1 \to \frac{1}{4} \cdot \pi^2$

$VOXMx := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx$ $VOXMx \to \frac{1}{16} \cdot \pi^3 - \frac{1}{4}$ $VOXx_{average} := \frac{VOXMx}{VOXM1}$ $VOXx_{average} \to 4 \cdot \frac{1}{16}$

$VOYM1 := \int_a^b 2\pi \cdot x \cdot (f2(x) - f1(x))\, dx$ $VOYM1 \to \pi^2 - 2 \cdot \pi$

$VOYMy := \int_a^b \pi \cdot x \cdot \left(f2(x)^2 - f1(x)^2\right) dx$ $VOYMy \to \frac{1}{16} \cdot \pi^3 - \frac{1}{4}$ $VOYy_{average} := \frac{VOYMy}{VOYM1}$ $VOYy_{average} \to \frac{\frac{1}{16} \cdot \pi}{\pi^2}$

$$\frac{1}{4}\cdot\exp(2) + \frac{1}{4}\Big)^2$$

$$\frac{\exp(2) - \frac{1}{4}\cdot\pi}{\mathrm{xp}(1) - 2\cdot\pi}$$

$$\frac{\exp(2) - \frac{1}{4}\cdot\pi}{\exp(2) + \frac{1}{2}\cdot\pi}$$

?)

2)

$$\left(\cdot 1\right)^{2}$$

$$\frac{\cdot \pi^{3} - \frac{1}{4} \cdot \pi}{\pi^{2}}$$

$$\frac{^{3} - \frac{1}{4} \cdot \pi}{- 2 \cdot \pi}$$