

Limits in symbolic form

Limits by definition

$$f(x) := x^3$$

$$\lim_{\Delta x \rightarrow 0^+} \frac{f(x + \Delta x) - f(x)}{\Delta x} \rightarrow 3 \cdot x^2$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \rightarrow 3 \cdot x^2$$

$$\lim_{\Delta x \rightarrow 0^+} \frac{(x + \Delta x)^a - x^a}{\Delta x} \rightarrow x^a \cdot \frac{a}{x}$$

$$\lim_{\Delta x \rightarrow 0^+} \frac{b^{x+\Delta x} - b^x}{\Delta x} \rightarrow b^x \cdot \ln(b)$$

Right and Left Limits

$$f(x) := |x - 2|$$

$$\lim_{\Delta x \rightarrow 0^+} \frac{f(2 + \Delta x) - f(2)}{\Delta x} \rightarrow 1$$

$$\lim_{\Delta x \rightarrow 0^-} \frac{f(2 + \Delta x) - f(2)}{\Delta x} \rightarrow -1$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x} \rightarrow \text{undefined}$$

Rational Limits

$$\lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x^2 - 5x + 6} \rightarrow -2 = -2$$

$$\lim_{x \rightarrow \infty} \frac{x^3 - 4 \cdot x^2 - x + 1}{4 \cdot x^3 - 5 \cdot x + 7} \rightarrow \frac{1}{4} = 0.25$$

IrRational Limits

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - x^{12}}}{1 - \sqrt{1 - 2 \cdot x^{12}}} \rightarrow \frac{1}{2}$$

$$\lim_{x \rightarrow 1} \frac{1 - \sqrt[6]{x}}{1 - \sqrt[3]{x}} \rightarrow \frac{1}{2}$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{2n^4 + n^2 + 1} - \sqrt{2n^4 - n^2 + 1} \right) \rightarrow \frac{1}{2} \cdot 2^{\frac{1}{2}}$$

$$\lim_{n \rightarrow \infty} n \cdot \left(\sqrt{n^4 + n^2 + 3n + 1} - \sqrt{n^4 + n^2 - 3n + 1} \right) \rightarrow 3$$

$$\lim_{n \rightarrow \infty} n^2 \cdot \left(\sqrt{n^4 + n^2 + 5} - \sqrt{n^4 + n^2 - 5} \right) \rightarrow 5$$

Trigonometric Limits

$$\lim_{x \rightarrow 0} \frac{\sin(3 \cdot x) - 3 \sin(x)}{x} \rightarrow 0$$

$$\lim_{x \rightarrow 0} \frac{\sin(3 \cdot x) - 3 \sin(x)}{x^2} \rightarrow 0$$

$$\lim_{x \rightarrow 0} \frac{\sin(3 \cdot x) - 3 \sin(x)}{x^3} \rightarrow -4$$

Exponential Limits

$$\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} \rightarrow \exp(1)$$

$$\lim_{x \rightarrow 0^+} (1 + x)^{\frac{1}{x}} \rightarrow \exp(1)$$

$$\lim_{x \rightarrow 0^+} e^{\frac{1}{x}} \rightarrow \infty$$

$$\lim_{x \rightarrow 0^-} e^{\frac{1}{x}} \rightarrow 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{n^n} \rightarrow 1$$

Die Hard

$$\lim_{x \rightarrow e} \frac{e^x - x^e}{(x - e)^2} \rightarrow \frac{1}{2} \cdot \exp(-1 + \exp(1)) = 2.787471$$